HYPERBOLA

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 B

$4x^2 - 9y^2 - 8x = 32$

$\Rightarrow 4(x-1)^2 - 9y^2 = 36 \Rightarrow \frac{(x-1)^2}{9} - \frac{y^2}{4} = 1$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{4}{9} = \frac{13}{9} \Rightarrow e = \frac{\sqrt{13}}{3}$

Sol.2 D

$\sqrt{3}\,x - y - 4\sqrt{3}\,k = 0$ ...(1)

$\sqrt{3}\,kx + ky - 4\sqrt{3} = 0$ ...(2)

Solve (1) and (2)

$x = 2\frac{(1+k^2)}{k}$ and $y = \frac{2\sqrt{3}(1-k^2)}{k}$

$\frac{x^2}{4} - \frac{y^2}{12} = 4 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$ Hyperbola

Sol.3 A

$\frac{2b^2}{a} = 8$; $e = \frac{3}{\sqrt{5}}$

$\Rightarrow b^2 = 4a$; $e^2 = \frac{9}{5}$

$1 + \frac{b^2}{a^2} = \frac{9}{5} \Rightarrow \frac{b^2}{a^2} = \frac{4}{5}$

$\Rightarrow a = 5 \quad \Rightarrow b^2 = 20$

Hyp. $\frac{x^2}{25} - \frac{y^2}{20} = 1 \Rightarrow 4x^2 - 5y^2 = 100$

Sol.4 C

$C(0, 0)$ $A_1(4, 0)$ $F_1(6, 0)$

$CA_1 = 4$ $CF_1 = 6$

$\Rightarrow a = 4$ $ae = 6$

$a^2e^2 = 36 \quad \Rightarrow a^2\left(1+\frac{b^2}{a^2}\right) = 36$

$\Rightarrow b^2 = 36 - 16 \Rightarrow b^2 = 20$

Hyp. $\frac{x^2}{16} - \frac{y^2}{20} = 1$ or $5x^2 - 4y^2 = 80$

Sol.5 A

$F_1(6, 5)$ $F_2(-4, 5)$ $e = \frac{5}{4}$

$F_1F_2 = 2ae$ Centre of hyp. is the mid point of $F_1F_2 = (1, 5)$

$2ae = 10$

$\Rightarrow ae = 5 \Rightarrow a^2e^2 = 25 \Rightarrow a^2\left(\frac{25}{16}\right) = 25$

$\Rightarrow a^2 = 16 \Rightarrow b^2 = 9$

Hyp. $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

Sol.6 B

Centre of hyp. will be

mid point of A_1 & A_2 = $\left(\frac{10+0}{2}, 0\right)$

$= (5, 0)$

Sol.7 C

$2a = 7 \Rightarrow a = \frac{7}{2}$

Let the Equation of hyp.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

passes through $(5, -2)$

$\frac{25}{a^2} - \frac{4}{b^2} = 1$

$\frac{25}{a^2} - 1 = \frac{4}{b^2}$

$b^2 = \frac{4a^2}{25 - a^2} = \frac{4 \times \frac{49}{4}}{25 - \frac{49}{4}} = \frac{196}{51}$

Equation $\frac{4x^2}{49} - \frac{51y^2}{196} = 1$

Sol.8 **B**

$x^2 - y^2 \sec^2\alpha = 5$

$$\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1 \to e_1$$

$$e_1 = 1 + \frac{5\cos^2\alpha}{5} = 1 + \cos^2\alpha \quad ...(1)$$

$x^2 \sec^2\alpha + y^2 = 25$

$$\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1 \to e_2$$

$$e_2 = 1 - \frac{25\cos^2\alpha}{25} = 1 - \cos^2\alpha$$

$e_1 = \sqrt{3}\ e_2$

$e_1{}^2 = 3e_2{}^2$

$1 + \cos^2\alpha = 3 - 3\cos^2\alpha$

$4\cos^2\alpha = 2$

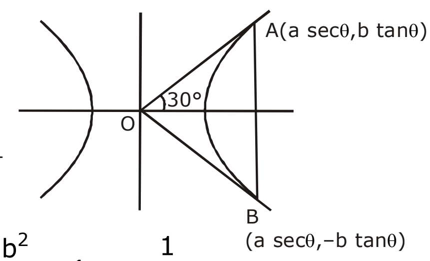
$$\cos\alpha = \frac{1}{\sqrt{2}} \quad \Rightarrow \alpha = \frac{\pi}{4}$$

Sol.9 $\theta = 30°$

$$\frac{b\tan\theta}{a\sec\theta} = \tan 30°$$

$$\frac{b}{a}\sin\theta = \frac{1}{\sqrt{3}}$$

$$\frac{b}{a} = \frac{1}{\sqrt{3}\sin\theta}$$



$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{1}{3\sin^2\theta}$$

$$e^2 > 1 + \frac{1}{3}$$

$$e > \frac{2}{\sqrt{3}}$$

Sol.10 **B**

$$\frac{x^2}{18} - \frac{y^2}{9} = 1$$

Slope of tangent would be = –1

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = -x \pm \sqrt{3}$

Sol.11 **C**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Tangent

$y = mx \pm \sqrt{a^2m^2 - b^2} \quad ...(1)$

$$\frac{x^2}{(-b^2)} - \frac{y^2}{(-a^2)} = 1$$

$y = mx \pm \sqrt{(-b^2)m^2 + a^2} \quad ...(2)$

(1) and (2) are same

$$\frac{1}{1} = \frac{1}{1} = \frac{\sqrt{a^2m^2 - b^2}}{\sqrt{a^2 - b^2m^2}}$$

$a^2 - b^2 m^2 = a^2m^2 - b^2$

$m^2 = 1 \Rightarrow m = \pm 1$

$y = \pm x \pm \sqrt{a^2 - b^2}$

Sol.12 **D**

Locus of the feet of the $\perp^n$ drawn from any focus of the the hyp. upon any tangent is its auxilary circle

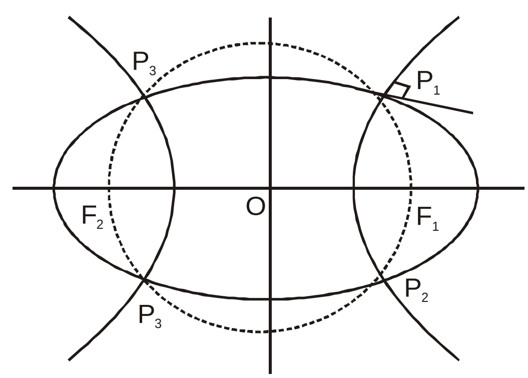
$$\text{Hyp. } \frac{x^2}{\left(\frac{1}{16}\right)} - \frac{y^2}{\left(\frac{1}{9}\right)} = 1$$

Auxiliary circle $x^2 + y^2 = \dfrac{1}{16}$

Sol.13 **A**

If they intersect at right angles then circle will pass through its focus

Circle will be



$x^2 + y^2 = (OF_1)^2$

$x^2 + y^2 = (\sqrt{5})^2$

$x^2 + y^2 = (\sqrt{5})^2 \quad ; \quad F_1(ae, 0) \quad e = \sqrt{5}$

$x^2 + y^2 = 5 \quad ; \quad F_1(\sqrt{5}, 0)$

Sol.14 A

Tangent to the parabola

$y = mx + \frac{2}{m}$ ...(1)

Tangent to the Hyp.

$y = mx \pm \sqrt{m^2 - 3}$ ...(2)

(1) and (2) are same $1 = \frac{2}{m\sqrt{m^2 - 3}}$

$m^2 - 3m^2 - 4 = 0 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

From (1) $2x \pm y + 1 = 0$

Sol.15 B

Slope of the chord $= \frac{25}{16} \times \frac{x_1}{y_1}$

$= \frac{25}{16} \times \frac{6}{2} = \frac{75}{16}$

Equation of chord passing through (6, 2)

$y - 2 = \frac{75}{16}(x - 6)$

$16y - 32 = 75x - 450$

$75x - 16y = 418$

Sol.16 A

Hyp. $xy - 3x - 2y = 0$

$f(x, y) = xy - 3x - 2y$

$\frac{\delta f}{\delta x} = 0 \Rightarrow y = 3$

$\frac{\delta f}{\delta y} = 0 \Rightarrow x = 2$ Centre (2, 3)

Asy. $xy - 3x - 2y + C = 0$

will pass through (2, 3)

$C = 6$

$xy - 3x - 2y + 6 = 0$

$(y - 3)(x - 2) = 0$

$x - 2 = 0,\ y - 3 = 0$

Sol.17 B

Hyp. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Let the point $P(a\sec\theta, b\tan\theta)$

Asy $y = \pm \frac{b}{a}x$

$ay - bx = 0$ and $ay + bx = 0$

$p = p_1 \cdot p_2$

$= \left|\frac{ab\tan\theta - ab\sec\theta}{\sqrt{a^2 + b^2}}\right| \left|\frac{ab\tan\theta + ab\sec\theta}{\sqrt{a^2 + b^2}}\right|$

$p = \frac{a^2b^2}{a^2 + b^2}$

$\Rightarrow \frac{a^2b^2}{a^2 + b^2} = 6$ ....(1)

$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2}$

$\Rightarrow a^2 + b^2 = 3a^2$ ....(2)

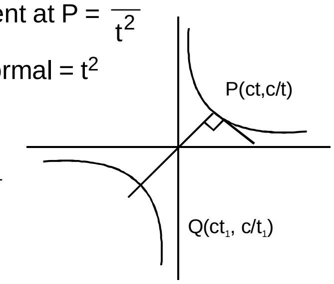
(1) and (2) $b^2 = 18$

$\Rightarrow a^2 = 9 \Rightarrow a = 3 = TA = 2a = 6$

Sol.18 B

Slope of tangent at P $= \frac{-1}{t^2}$

So slope of normal $= t^2$



$t^2 = \frac{\frac{c}{t_1} - \frac{c}{t}}{(ct_1 - ct)}$

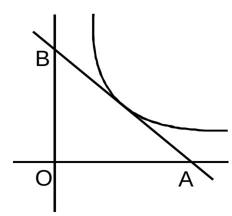
$t^2 = \frac{-1}{t_1 t}$

$t^3 t_1 = -1$

Sol.19 Tangent to the hyp. $xy =- c^2$

$\frac{x}{x_1} + \frac{y}{y_1} = 2$ (16, 1)

$\frac{x}{16} + \frac{y}{1} = 2$



$x + 16y = 32$

A(32, 0)

B(0, 2)

Area $= \frac{1}{2} \times 2 \times 32 = 32$ Sq. unit

Sol.20 A

Let the middle point M(h, k)

chord $T = S_1$ $xy = c^2$

$\frac{x}{h} + \frac{y}{k} = 2$

Slope $= \frac{-k}{h}$

$-\frac{-k}{h} = m$

$mh + k = 0$

$mx + y = 0$